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Scattering of positron by hydrogen-like ion

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Abstract Scattering of positron by hydrogen-like ion He^+ has been theoretically analysed by employing distorted-wave method. Total cross section and differential cross sections for $1s$-$2s$ excitation of He^+ by positron-impact, have been evaluated in intermediate and high energy ranges. Differential cross sections have been calculated with polarization potential and without polarization potential to observe the impact of polarization potential on differential cross section. Phase shifts for e^+-He^+ elastic scattering have been calculated. Results have been compared with available theoretical results.

Keywords Scattering of positron, hydrogen-like ion, distorted wave method, phase shift.

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It is interesting to study the behaviour of positrons in presence of the long-range Coulomb potential and attractive polarization potential at different projectile energies because it helps in analysis and interpretations of interactions associated with plasmas and cosmic rays.

Coulomb-Born and Coulomb-Born-Oppenheimer approximations were employed by Mitra and Sil [1] in evaluating excitation cross sections for e^--hydrogen-like ions. Shimamura [2] employed Harris-variational method to investigate positron-hydrogen-like ions scattering and reported the phase shifts for elastic positron - He^+ scattering. Khan *et al* [3] employed two variants of the polarized orbital method and reported the phase shifts for the elastic positron - He^+ scattering. Glauber approximation and modified Glauber approximation have been employed by Gien [4] in investigating the excitation of He^+ by electron impact at intermediate scattering energies.

In intermediate and high energy ranges, we have employed the variant of distorted wave method that accounts for the effect of continuum and effect of higher excited states. In this approach, we included a part of Coulomb potential in continuum wave function in the initial channel. The nucleus of the target has been taken as screened nucleus. The dipole polarization potential has been included which accounts for excited states partially. This approach has been earlier employed in the analysis of scattering of positrons by complex atoms with due success.

The Hamiltonian H for positron-hydrogen-like ion can be expressed as follows :

$$H = \left[-\frac{1}{2}\nabla_{r_1}^2 - \frac{1}{2}\nabla_{r_2}^2 - \frac{1}{r_1}\right] + \left[\frac{1}{r_2} - \frac{1}{r_{12}}\right] \tag{1}$$

In centre of mass reference frame, $\bar{r}_1$ and $\bar{r}_2$ are position vectors of atomic electron and incident positron with respect to the screened nucleus of target atom. $\nabla_{r_1}^2$ and $\nabla_{r_2}^2$ are kinetic energy operators. Atomic units have been used throughout.

The Hamiltonian H is partitioned and expressed as

$$H = H_i + W_i \tag{2}$$

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where $H_1 = H_0 + U$; $U = \frac{\delta}{r_2}$; $W = \frac{(1-\delta)}{r_2} \quad r_{12}$

δ = screening parameter and

H_0 = Free Hamiltonian.

The functions φ_r, χ_r and ψ_r satisfy the following Schrodinger equations with Hamiltonian H_0, H_1 and H, respectively.

$$\left.\begin{aligned} H_0\varphi_r &= E_r\varphi_r , \\ H_1\chi_r &= E_r\chi_r , \\ H_1\chi_r &= E_r\chi_r . \end{aligned}\right\} \tag{3}$$

Let the system move from initial bound state $|i>$ with momentum $\bar{K}_i$ to the final state $|f>$ with momentum $\bar{K}_f$. Assuming W to be a weak potential, T-matrix element can be expression as follows :

$$T_{i\to f} = \left\langle \varphi_f |V| \chi_r^{(+)} \right\rangle , \tag{4}$$

where $V = \frac{1}{r_2} - \frac{1}{r_{12}}$.

In atomic units differential cross section is expressed as

$$\frac{d\sigma}{d\Omega} = \frac{\mu^2}{4\pi^2}\frac{K_f}{K_i}|T|^2 \tag{5}$$

and total cross section is obtained by integrating the above expression

$$\sigma = \int\left(\frac{d\sigma}{d\Omega}\right)d\Omega . \tag{6}$$

Here wave functions have been taken in the following forms

$$\varphi_r = \exp\left(i\bar{k}.\bar{r}_2\right)\varphi_{nlm}(r_1) , \tag{7}$$

$$\chi_r = \Gamma(1+ia)\exp\left(i\bar{k}.\bar{r}_2 - \pi a/2\right) ,$$

$$x_1 F_1\left(-ia;1;ik_i r_2 - i\bar{k}_i r_2 - i\bar{k}_2\bar{r}_2\right)\varphi_{nlm}(\bar{r}_1) , \tag{8}$$

where $a = \frac{-}{k}$ and $\bar{k}$ is the momentum for the direct channel in which distortion is introduced. This method includes a part of Coulomb potential in incident continuum wave function. The effect of the continuum is taken into account partially. The radial part of wave function for hydrogen-like ion is expressed as

$$\varphi_n(r) = \sum a_i r_i^{n-1}\exp(-b_i r) . \tag{9}$$

For ground state, $n = 1$; $a_1 = \frac{Z^{3/2}}{\sqrt{\pi}}$; $b_1 = Z$.

For $2s$ state, $n = 2$; $a_1 = \frac{Z^{3/2}}{2\sqrt{2\pi}}$; $b_1 = \frac{Z}{2}$;

$$a_1 = -\left(\frac{Z}{2}\right)a_1 ; \quad b_2 = \frac{Z}{2} .$$

Now, T-matrix can be expressed as follows

$$T_{i\to f} = MF'\left[N_1\frac{\partial I}{\partial\lambda} + N_2\frac{\partial^2 I}{\partial\lambda^2} \right] \tag{10}$$

In above expressions, $\lambda = \frac{3Z}{2}$;

$$MF' = \Gamma(1+ia)\exp(-\pi a/2) ; \quad N_1 = -\frac{4\pi Z^3}{2\sqrt{2\pi}}\left(\frac{2Z}{\lambda^3} - \frac{}{\lambda^2}\right)$$

and $\quad N_2 = 4\pi\frac{Z^3}{2\sqrt{2\pi}}\left(\frac{Z/2}{\lambda^2}\right)$.

$$I = \iint \exp\left(i\bar{q}.\bar{r}_2\right)_1 F_1\left(-ia;I;ik_i r_2 - i\bar{k}_i.\bar{r}_2\right)\frac{e^{-\lambda r_1}}{r_{2/3}}dr_1 dr_2$$

Using Nordsieck-method [5], the integrals involved in differential cross sections are evaluated.

Polarization potential :

$$V_p(x) = -\frac{9}{4x^2}\left[1 - e^{-2x}\left(1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2}{3}x^4 + \frac{4}{27}x^5\right)\right]$$

where $x = Zr^2$.

This is the form of polarization potential used in the present analysis of scattering. The differential cross sections have been calculated without polarization (DCSWP) and with polarization (DCSPP) to analyse the impact of interaction on scattering of positrons.

Phase shifts :

In case of positron scattering, the radial wave function satisfy following equations:

$$\left[\frac{d}{dx^2} - \frac{1(1+1)}{x^2} + K^2\right]g_1(x) = 0 , \tag{13}$$

$$\left[\frac{d}{dx^2} - \frac{1(1+1)}{x^2} + V(x) + K^2\right]u_1(x) = 0 . \tag{14}$$

In the above expressions, x is the coordinate of incident positron, $V(x)$ is the sum of static potential and polarization potential, $g_1(x)$ and $u_1(x)$ are the components of radial wave function such that they vanish at $x = 0$.

In intermediate and high energy ranges, the potential has been taken to be weak potential and approximate solutions have been obtained and phase shifts have been evaluated. Precautions are taken to avoid divergence. The evaluated phase shifts are compared with available theoretical data.

The total cross sections for $1s$–$2s$ excitations of He^+ have been calculated for the intermediate and high energy range employing distorted wave method without polarization potential (TCSWP) and with polarization potential (TCSPP). The curve F represents the values corresponding to TCSWP whereas curve A represents those of TCSPP. The curve A lies below the curve F. This displays the impact of polarization potential on total cross sections, which is prominent at low projectile energies.

The curves B and C show the total cross sections for $1s$–$2s$ excitation of He^+ by electron impact, calculated by Gien [4] employing Glauber approximation and modified Glauber approximation whereas curve D represents the total cross sections for $1s$–$2s$ excitation of He^+ by electron impact, using Coulomb-Born-Oppenheimer approximation.

The static interaction which arises from the Coulomb field of the undistorted atom is attractive for electron scattering and repulsive for positron scattering but it contributes to positron scattering at all projectile energies. The exchange interaction is dominant at low energies. The polarization interaction is attractive for both electrons and positrons and is important at low projectile energies. The combined impact of static and the static and polarization interactions is that for electron scattering they add to each other, while for positron scattering, there is a tendency for cancellation. Therefore, at low projectile energies,

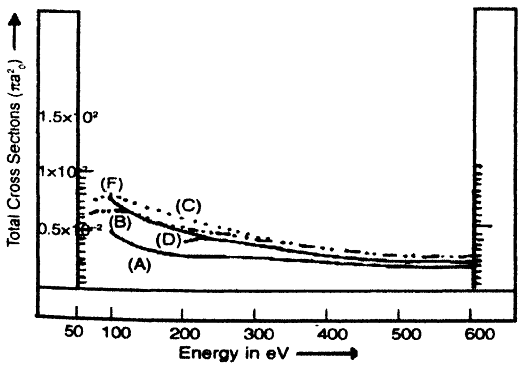

Figure 1. $1s - 2s$ Excitation of He^+ by Positron Impact. A -Distorted wave approximation with polarization, B-$1s$–$2s$ excitation of He^+ by electron impact in Glauber approximation [4], C-$1s$–$2s$ excitation of He^+ by electron impact in modified Glauber approximation [4] Coulomb-Born-Oppenheimer [1], F-Distorted wave approximation without polarization.

it is expected that total cross sections for positrons are lower than those for electrons. This feature is clearly displayed in Figure 1, curve A lies below B, C and D at low energies of incident positrons.

This shows that present approach is a suitable one.

With rise in the energy of incident positrons, the curve A is getting closer to curve D and B and at high energies they come very close to each other. They have the tendency of merging together at higher incident energy of positrons. This merging tendency is displayed in Figure 1, which was expected. This observation shows that the present method is suitable for theoretical analysis of positron - atom scattering.

It is well known that the polarization and exchange interactions become negligible at higher energies of projectile and static interaction plays an important role. Hence, it is expected that total cross sections for positrons and for electrons merge together. This feature is clearly displayed in Figure 1. Thus present method of analysis is well justified.

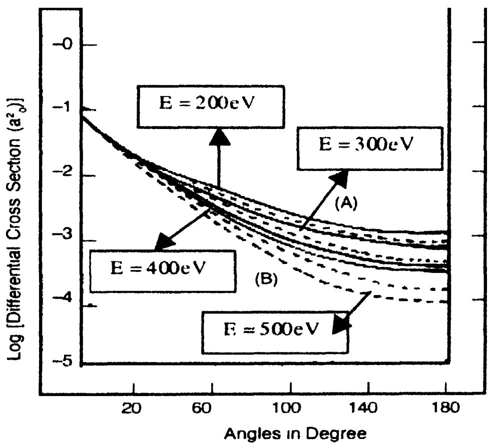

Figure 2. $1s - 2s$ Excitation of He+ by positron impact , A - Distorted wave without polarization __ _________, B -Distorted wave with polarization ____________

Differential Cross Sections :

Before evaluation of total cross sections for $1s - 2s$ excitation of He^+ by positron impact, differential cross sections have been calculated by employing distorted wave method without polarization potential (DCSWP) and with polarization potential (DCSPP) in the intermediate and high energy ranges. The curve A represents the values of DCSWP whereas curve B represents those of DCSPP. The Figure 2 shows the variation of differential cross sections with angle of scattering at 200 eV, 300 eV , 400 eV and 500 eV.

At all projectile energies, curve B lies below the curve A, this exhibits the impact of polarization interaction. Moreover, Figure 2 makes it evident that all curves have tendency of merging

Table 1. Phase shifts.

Energy in eV	η_p	η_C	η_{TL}
s-wave phase shifts (rad) for e^+ - He^+ elastic scattering -			
15 688	0.356 (-2)	0 49 (-2)	0.79 (-2)
18 440	0.230 (-2)	0.22 (-2)	0.63 (-2)
21 703	-0 26 (-2)	-0.25 (-2)	0 27 (-2)
29 716	-1.90 (-2)	-1 90 (-2)	-1.07 (-2)
30 929	-2 17 (-2)	-2.19 (-2)	-1.31 (-2)
37 514	-3 02 (-2)	-3.79 (-2)	-2 68 (-2)
p-wave phase shfts (rad) for e^+-He^+ elastic scattering.			
15 688	0.640 (-2)	0 63 (-2)	0.71 (-2)
18 440	0 740 (-2)	0 73 (-2)	0 83 (-2)
21 703	0.803 (-2)	0.80 (-2)	0 93 (-2)
29 716	0.797 (-2)	0 80 (-2)	1 00 (-2)
30.929	0 720 (-2)	0.78 (-2)	0.99 (-2)
37 514	0.577 (-2)	0.62 (-2)	0.88 (-2)
d-wave phase shifts (rad) for e^+-He^+ elastic scattering			
15 688	0 262 (-2)	0 31 (-2)	0.33 (-2)
18.440	0 380 (-2)	0 37 (-2)	0.41 (-2)
21.703	0 519 (-2)	0.45 (-2)	0 43 (-2)
29.716	0 581 (-2)	0.59 (-2)	0 66 (-2)
30.929	0 607 (-2)	0.60 (-2)	0 68 (2)
37.514	0.632 (-2)	0.68 (-2)	0 77 (-2)

η_p = Present value phase shifts ,
η_C = Phase shift with potential of Callaway [6]
η_{TL} = Phase shift with the potential of Temkin and Lamkin [7]

together closer to forward angle, which is expected. This justifi the suitability of present method of analysis of positro scattering.

Phase shifts for e^+ - He+ elastic scattering :

Table 1 represents *s*-wave, *p*-wave and *d*-wave phase shifts

As literature reveals, the effect of positronium formatio channel on the elastic channel is negligible; so in prese theoretical investigation, the effect of positronium formatio has been neglected. The present phase shifts have bee compared with corresponding two sets of phase shifts predict by Khan *et al* [3] who employed two variants of polarized orbit method, η_c and η_{TL} are phase shifts evaluated by abo investigators with potentials of Callaway [6] and of Temkin a Lamkin [7], respectively.

The present set of results show good agreement wi corresponding values of η_c and slight difference with η_{T} This slight difference may be due to diabatic potential which repulsive in nature and add to the repulsive static potential Th observed agreement establish the fact that present approach good one and can be used in further theoretical investigatio of position-atom scattering.

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